Abstract—The introduction of active devices in Smart Grids, such as smart transformers, powered by intelligent software and networking capabilities, brings paramount opportunities for online automated control and regulation. However, online mitigation of disruptive events such as cascading failures, is challenging. Local intelligence by itself cannot tackle such complex collective phenomena with domino effects. Collective intelligence coordinating rapid mitigation actions is required. This paper introduces analytical results from which two optimization strategies for self-repairable Smart Grids are derived. These strategies build a coordination mechanism for smart transformers that runs in three healing modes and performs collective decision-making of the phase angles in the lines of a transmission system to improve reliability under disruptive events, i.e. line failures causing cascading failures. Experimental evaluation using self-repairability envelopes in different case networks, AC power flows and varying number of smart transformers confirms that the higher the number of smart transformers participating in the coordination, the higher the reliability and the capability of a network to self-repair.

Index Terms—smart transformer, optimization, coordination, cascading failure, reliability, repairable network, self-repairability envelope

I. INTRODUCTION

The introduction of Information and Communication Technologies (ICT) in traditional power systems has brought phenomenal opportunities for online automated control and decentralized self-regulation in Smart Grids. These new capabilities can increase the integration of renewable energy resources, reduce the operational costs and improve the reliability of power systems under highly disruptive complex phenomena such as cascading failures. Several electrical devices [1], [2] powered by intelligent software play a key role in this new era, e.g. smart transformers [3]. In 2011, smart transformer was chosen by MIT an one of the ten emerging technology breakthroughs that can have the greatest impact in the world [4]. However, a smart transformer by itself cannot shape the future self-repairable Smart Grids. Online coordination of all system components is required to prevent or rapidly respond to disruptive events such as cascading failures and cyber-attacks. This paper contributes a method for online coordination of multiple smart transformers so that their synergistic control of power flow prevents a cascading failure or minimizes its impact when it occurs.

This paper studies the reliability of highly meshed transmission systems with phase shift transformers. Multiple transformers provide redundancy as in case of a single transformer failure, operational flexibility is guaranteed by the remaining ones. The phase angle of the transformers governs the overall power flow distribution in the network and is usually determined via day-ahead operational planning based on forecasts. Decision-making is usually performed offline by system operators [5]. Manual online adjustments can be performed involving coordination with telephone conversations of up to 15 minutes duration [6]. However, the penetration of renewable energy resources, the exchange of flow between regions or even the highly dynamic demand originated from demand-response programs [7] and micro-generation capabilities require an almost real-time and fully automated regulation of phase shift transformers. Regulating the flow of a transmission system with multiple phase shift transformers is challenging as coordination is required given the non-linear dynamics of AC power flow networks. However, the cost-benefits of using coordinating phase shift transformers are well documented in earlier work [8]. As a cascading failure involves several network components, coordinating phase shift transformers is highly applicable in this context and scenarios.

This paper introduces a model of self-repairable Smart Grids using smart transformers. In this paper, a self-repairable Smart Grid refers to the inner system capability to prevent or mitigate failures, such as cascading ones, in an online and automated way using its own cyber-physical assets such as smart transformers. A smart transformer here is defined by a phase shift transformer running software that controls the phase angle and can remotely communicate with other assets of the network running such a software. The proposed model brings together three capabilities when a disruptive event, such as a line failure, occurs: (i) load shedding, (ii) generation balancing and (iii) optimization of flow distribution via coordination of smart transformers. Three healing modes of smart transformers are evaluated in which smart transformers operate at different stages of a cascading failure. This paper contributes analytical expressions that determine the flow in the power lines as a function of the phase angles in the smart transformers. Coordination of the smart transformers is achieved with two optimization strategies evaluated in various reference networks and loading profiles. Results show that the two strategies improve the system reliability by decreasing the load shedded and the lines trimmed. The coordination mechanism can increase the reliability further if more smart transformers participate in the online coordination process.

The main contributions of this paper are summarized as
follows:

- The use case of smart transformers for mitigating cascading failures.
- A coordination algorithm of smart transformers operating in three healing modes against cascading failures that minimizes load-shedding and bounds generation limits.
- Analytical expressions for both AC and DC flow models for the regulation of flows using smart transformers.
- The envelopes of self-repairability. They are introduced as an evaluation methodology to assess the overall resilience of a power network against cascading failures using smart transformers.

This paper is outlined as follows: Section II illustrates a model for self-repairable Smart Grids using two strategies and three heater modes of smart transformers. Section III illustrates analytical expressions for the coordination of smart transformers. It also introduces two optimization strategies that improve the repairability of Smart Grids under N-1 contingency analysis. Section IV evaluates the proposed coordination approach in different networks, AC power flows and varying number of smart transformers. Section V compares the proposed model of self-repairable Smart Grids with related work. Finally, Section VI concludes this paper and outlines future work.

II. SELF-REPAIRABLE SMART GRIDS

This section introduces a model for online repairable Smart Grids under perturbations that influence the flow of the power lines. Such perturbations may include failure of power lines and/or changes of power load and generation. The goal of the model is to mitigate such disruptive events by minimizing their impact until the system returns to its stable state via, for example, manual offline system repair and maintenance. The model can be used for operational planning, as well as real-time mitigation if computational resources are acquired for this purpose. It is applicable to power transmission networks, though the computational optimization methods employed in this paper are relevant for the reliability of power distribution networks as well. Figure 1 shows a high-level illustration of the proposed model for self-repairable Smart Grids.

![Diagram](image-url)

Fig. 1. A model for self-repairable Smart Grids using coordinating smart transformers.

The model can be realized as a control system shaped over event detection and islanding management. The model requires the detection of an event that disturbs the balance of supply and demand, e.g. the failure of a power line. Moreover, in case the event disconnects the network resulting in multiple islands, the control system is applied to each formed island.

The sensing part of the model includes the computation of the DC or AC flow in the power lines given (i) the physical characteristics of the network, (ii) constraints in generator limits and (iii) control actions performed by the coordinating smart transformers. Sensing indicates if the system configuration converges, meaning a solution is found that balances supply and demand.

The control part includes the algorithmic logic for actuation given information about the DC/AC power flow. It also concerns the coordination of smart transformers by collectively choosing the phase angles of several phase shift transformers to mitigate flow imbalances. Coordination is performed using two optimization strategies derived from analytical results. They are referred to as STRATEGY 1 and STRATEGY 2. Section III illustrates the analytical results and the strategies in detail.

The actuation part concerns (i) load shedding, (ii) generation balancing, (iii) phase angle shifting and (iv) line trimming. Load shedding reduces the level of the loads in the network given the computations performed in the control part. Load shedding also indicates whether there is a blackout by using the maximum number of iterations or the continuation power flow threshold as possible criteria. Generation balancing sets the operation of slack buses within the range of their minimum and maximum capacity. These physical constraints are usually not determined in the power flow analysis. The coordinated shift of the phase angles is computed in the control part and applied in the actuation part. Finally, line trimming disconnects overloaded power lines to prevent their physical damage.

Algorithm 1 illustrates the healing operations performed for improving the Smart Grid reliability. The algorithm is illustrated for AC power flow. It can be simplified and adjusted for DC power flow as well, given that flow convergence can always be satisfied. Load shedding is a mitigation countermeasure applied when flow does not converge (lines 18–24 in Algorithm 1). There is no universal standardized load reduction strategy among different systems regulators, e.g. ENTOSE-3, NERC, etc. Some possible load-shedding algorithms are outlined in earlier work [10]. Generation limits are met (lines 9–14 in Algorithm 1) by repeating the following process: when the flow of an existing slack buses surpasses its limits, the flow is set to its maximum, if flow is positive, or minimum, if flow is negative. The process repeats by selecting another generator with the highest maximum power suggesting high inertia.

A system that mitigates a disruptive event with load-shedding and generation balancing is referred to as BASE CASE, in contrast to STRATEGY 1 and STRATEGY 2 that additionally employ coordinating smart transformers. Smart transformers can operate in three healing modes under AC flows: (i) HEALER 1 mitigates a cascading failure right after a system perturbation and before load shedding is applied (core stage). HEALER 1

1The inertia of a generator is proportional to its nominal power (maximum power) [11]. In real operation, the saturation of the power output, meaning in this case the generation limits, is given by primary and secondary frequency control contracts [12].
Algorithm 1 Event mitigation with smart transformers.

Require: detection of disruptive events
1: Early stage: run smart transformer coordination
2: for each disruptive event do
3: extract islands
4: for each island do
5: while line limits are violated do
6: loop
7: flow = power flow analysis
8: if flow is converged then
9: if limits are not violated then return
10: else
11: meet generator limits
12: end if
13: flow = power flow analysis
14: if load is not shedded then
15: end if
16: Core stage: run smart transformer coordination
17: end if
18: else if flow is not converged and is not blackout then
19: for iteration 1 to maximum do
20: load = shedded
21: flow = power flow analysis
22: if flow is converged then
23: return
24: else
25: end for
26: if iteration = maximum then
27: flow = blackout
28: end if
29: if flow is blackout then
30: return
31: end loop
32: end if
33: end else
34: Final stage: run smart transformer coordination
35: end while
36: end for

Table I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of buses</td>
</tr>
<tr>
<td>L</td>
<td>Number of lines</td>
</tr>
<tr>
<td>C</td>
<td>Incidence matrix of a directed network</td>
</tr>
<tr>
<td>f</td>
<td>Vector of active power flow in lines</td>
</tr>
<tr>
<td>f_L</td>
<td>The active power flow over a line ( \ell )</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Vector of phase shift angles</td>
</tr>
<tr>
<td>( H_{angle} )</td>
<td>Matrix of sensitivity factors for the phase shift angles</td>
</tr>
<tr>
<td>( B_{line} )</td>
<td>Line susceptance matrix, no reference &amp; slack buses</td>
</tr>
<tr>
<td>( B_{bus} )</td>
<td>Bus susceptance matrix, no reference &amp; slack buses</td>
</tr>
<tr>
<td>( I )</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>( b )</td>
<td>Vector of the series susceptance in lines</td>
</tr>
<tr>
<td>( \mathbf{1}^T )</td>
<td>Transposed vector of 1s</td>
</tr>
<tr>
<td>( b_\ell )</td>
<td>The series susceptance of line ( \ell )</td>
</tr>
<tr>
<td>( z_\ell^2 )</td>
<td>The series reactance of line ( \ell )</td>
</tr>
<tr>
<td>( B_{line} )</td>
<td>Line susceptance matrix</td>
</tr>
<tr>
<td>( B_{bus} )</td>
<td>Bus susceptance matrix</td>
</tr>
<tr>
<td>( \Delta f )</td>
<td>Vector of power flow transfers over a line ( \ell )</td>
</tr>
<tr>
<td>( \Delta p )</td>
<td>Vector of power flow transfers between buses</td>
</tr>
<tr>
<td>( \Delta \varphi )</td>
<td>Vector of shifts in line angles</td>
</tr>
<tr>
<td>( H_{bus} )</td>
<td>Matrix of sensitivity factors for bus power transfers</td>
</tr>
<tr>
<td>( H_{line} )</td>
<td>Matrix of sensitivity factors for line power transfers</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>Vector of initial power flow in lines</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Vector of phase shift angles in lines</td>
</tr>
<tr>
<td>( \varphi_0 )</td>
<td>Vector of initial phase shift angles in lines</td>
</tr>
<tr>
<td>( g_\ell )</td>
<td>Vector of phase shift distribution factor for line ( \ell )</td>
</tr>
<tr>
<td>( G_{angle} )</td>
<td>Matrix of phase shift distribution factors in DC</td>
</tr>
<tr>
<td>k</td>
<td>Number of smart transformers in a network</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Penalty parameter over control action</td>
</tr>
<tr>
<td>( \Delta \varphi )</td>
<td>The shift of the phase angle in line ( \ell )</td>
</tr>
<tr>
<td>( \Delta \varphi_{max} )</td>
<td>A maximum permitted shift of the angle in line ( \ell )</td>
</tr>
<tr>
<td>( \varphi_{max} )</td>
<td>A maximum value of the phase shift angle</td>
</tr>
<tr>
<td>( \alpha_\ell )</td>
<td>1/0 element of line ( \ell ) exceeding a threshold</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Vector of 1/0 line elements exceeding a threshold</td>
</tr>
<tr>
<td>( \beta_i )</td>
<td>Threshold ( i ) used in STRATEGY 2</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Number of thresholds used in STRATEGY 2</td>
</tr>
<tr>
<td>( w_1 )</td>
<td>Weight of vector ( \alpha_1 )</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>Load shedded</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Final load</td>
</tr>
<tr>
<td>( p )</td>
<td>Initial load</td>
</tr>
<tr>
<td>( L_t )</td>
<td>Trimmed lines</td>
</tr>
<tr>
<td>( L )</td>
<td>Final number of non-trimmed lines</td>
</tr>
<tr>
<td>( L_t )</td>
<td>Initial number of non-trimmed lines</td>
</tr>
<tr>
<td>U</td>
<td>Lines utilization</td>
</tr>
<tr>
<td>( r_\ell )</td>
<td>The rating of line ( \ell )</td>
</tr>
</tbody>
</table>

Theorem 1 provides the sensitivity factors for the phase shift angles of the power lines required to measure the power flow in the lines as a function of the angles. By using smart phase shift transformers to perform an actual control of the phase shift angles, the power flow in the lines can be optimized. This paper studies whether this flow optimization results in higher resilience under disruptive events such as cascading failures.

Theorem 1. The matrix of sensitivity factors for the phase shift angles in a transmission network with DC or AC power flow is given as follows:

\[
H_{angle} = (C \cdot (\tilde{B}_{line} \cdot \tilde{B}_{bus}^{-1})^T - I)(b \cdot 1^T),
\]

where \( C \) is the incidence matrix representing the transmission network as a directed graph, \( \tilde{B}_{line} \), \( \tilde{B}_{bus} \) is the line and bus susceptance matrices excluding the reference and slack buses.
Given the vector $b = [b_1, b_2, ..., b_L]^T$ of the series suscepcance in the lines.

**Proof.** Given the vector $b = [b_1, b_2, ..., b_L]^T$ of the series suscepcance in the lines, where each element involves the series reactance as $b_{\ell} = \frac{1}{\pi}$, the line suscepcance matrix can be expressed as follows:

$$B_{\text{line}} = (b \cdot 1^T)C.$$  \hfill (2)

The bus suscepcance matrix can be similarly derived as follows:

$$B_{\text{bus}} = C^T B_{\text{line}}.$$  \hfill (3)

If the reference and slack bus columns are excluded, the bus and line suscepcance matrices are referred to as $B_{\text{bus}}$ and $B_{\text{line}}$ respectively. The matrix of sensitivity factors for the power flow transfers $\Delta p$ between buses is defined as follows:

$$H_{\text{bus}} = \frac{\Delta f}{\Delta p} = B_{\text{line}} \cdot (B_{\text{bus}})^{-1}.$$  \hfill (4)

Similarly, the matrix of the sensitivity factors for the shift $\Delta \phi$ of phase angles can be expressed as follows:

$$H_{\text{angle}} = \frac{\Delta f}{\Delta \phi} = H_{\text{line}} (b \cdot 1^T).$$  \hfill (5)

Given that the matrix of the sensitivity factors for the power flow transfers between lines is given by $H_{\text{line}} = (C \cdot H_{\text{bus}}^T - I)$, it is derived that:

$$H_{\text{angle}} = (C \cdot (B_{\text{line}} \cdot (B_{\text{bus}})^{-1})^T - I) (b \cdot 1^T).$$  \hfill (6)

Given the initial power flow, and the matrix of sensitivity factors for the phase shift angles determined in Theorem 1, the power flow of lines after shifts in phase angles is derived in Corollary 1.

**Corollary 1.** The DC or AC power flow $f = [f_1, f_2, ..., f_L]^T$ of $L$ lines in a transmission network with phase angles $\phi = [\phi_1, \phi_2, ..., \phi_L]^T$ is computed as follows:

$$f(\phi) = f^0 + H_{\text{angle}} \Delta \phi,$$  \hfill (7)

where $f^0$ is the vector of the initial DC or AC power flow in the lines, $H_{\text{angle}}$ is the matrix of sensitivity factors for the phase angles and $\Delta \phi$ is the vector of shifts in the phase angles of the lines.

**Proof.** The power flow $f_{\ell}$ in a line $\ell$ can be written as a function of the phase angles in lines using a Taylor series expansion:

$$f_{\ell}(\phi) = \sum_{n=0}^{\infty} \frac{\partial^n f_{\ell}}{\partial \phi^n} \big|_{\phi=\phi_0} \Delta \phi^n,$$  \hfill (8)

where the vector of phase angles $\phi = \phi_0 + \Delta \phi$ is given by the vector of the initial phase angles $\phi_0$ and the vector with the shifts of the phase angles $\Delta \phi$. The series can be approximated in its first two terms as follows:

$$f_{\ell}(\phi) \approx f_{\ell}(\phi_0) + g_{\ell} \Delta \phi,$$  \hfill (9)

where $f_{\ell}(\phi_0)$ is the vector of power flow in the lines given the initial angles of the lines. The $g_{\ell}$ is a phase shift distribution factor and is represented by the vector of the derivatives for the power flow of line $\ell$ with respect to the vector of angles $\phi$ in the lines. It is computed as follows:

$$g_{\ell} = \frac{\partial f_{\ell}}{\partial \phi} \big|_{\phi=\phi_0}.$$  \hfill (10)

For a vector formulation, Equation 9 can be written as follows:

$$f(\phi) = f^0 + G_{\text{angle}} \Delta \phi,$$  \hfill (11)

given (i) the matrix approximation of the phase shift distribution factors $G_{\text{angle}} \approx \frac{\partial f}{\partial \phi}$ for a DC formulation in $\Delta$ changes, (ii) the vector of flows $f = [f_1, f_2, f_L]^T$ and (iii) $f(\phi_0) = f^0$. However, from Theorem 1, Equation 5 it holds that:

$$\frac{\Delta f}{\Delta \phi} = H_{\text{angle}} \approx G_{\text{angle}},$$  \hfill (12)

therefore it is proven that:

$$f(\phi) = f^0 + H_{\text{angle}} \Delta \phi.$$  \hfill (13)

Given Theorem 1 and Corollary 1, the smart operation and coordination of phase shift transformers can turn into a computational optimization problem for determining the phase angles of lines equipped with smart transformers. This section formulates two optimization strategies of power flow that assume a subset of $k \leq L$ lines equipped with phase shift transformers for influencing the phase angle. The transformers influence the phase angle of the line locally, however the flow distribution of the whole network may be influenced after such an action. The optimization strategies influence this flow redistribution such that a cascading failure is mitigated.

**Strategy 1.** The DC or AC power flow $f$ of $L$ lines in a transmission network can be optimized by minimizing the 2-norm as follows:

$$\min ||f||_2 + \lambda ||\Delta \phi||_2$$

subject to

$$f = f^0 + H_{\text{angle}} \Delta \phi$$

$$|\Delta \phi_{\ell}| \leq \Delta \phi_{\text{max}}, \forall \ell \in 1, ..., k$$

$$|\phi_{\ell}| \leq \phi_{\text{max}}, \forall \ell \in 1, ..., k$$

where $f$ is given by Corollary 1, $\lambda$ is the penalty parameter over the control action, $\Delta \phi_{\ell}$ is the phase shift of the phase angle in line $\ell$, $\phi_{\ell}$ is the phase angle of line $\ell$, $\Delta \phi_{\text{max}}$ is the maximum shift of phase angles and $\phi_{\text{max}}$ is the maximum value the phase shift angles can have.

The bounds of the angles and phase angle shifts can be chosen empirically by evaluating the power flow convergence. The magnitude of angle adjustments is regulated by the penalty parameter $\lambda$ applied over the control action. A version of Strategy 1 is used in earlier work [18], in contrast to Strategy 2 that is introduced here and is formulated as follows:
Strategy 2. The DC or AC power flow $f$ of $L$ lines in a transmission network can be optimized by minimizing the product of weighted power flows as follows:

$$
\min \sum_{i=1}^{v} \sum_{j=1}^{L} w_i a_{i,\ell} + \lambda \|\Delta \varphi\|_2
$$

subject to

$$
\begin{align*}
\Delta \varphi_{\ell} & \leq f^0 + H_{ang}\Delta \varphi \\
\varphi_{\ell} & \leq \varphi_{max}, \forall \ell \in 1, \ldots, k \\
a_{i,\ell} & = \begin{cases} 
1 & \text{if } |f_{\ell}| \geq \beta_i \\
0 & \text{otherwise}
\end{cases} \\
& \forall \ell \in 1, \ldots, L \\
& \forall i \in 1, \ldots, v
\end{align*}
$$

where $f$ is given by Corollary 1, $\lambda$ is the penalty parameter over the control action, $\Delta \varphi_{\ell}$ is the shift of the phase angle in line $\ell$, $\varphi_{\ell}$ is the phase shift angle of line $\ell$, $\varphi_{max}$ is the maximum shift of phase angles, $\varphi_{max}$ is the maximum value the phase shift angles can have, $a_{i,\ell}$ is the 1 or 0 element of line $\ell$ from the vector $a_i$ that indicates if the power flow $f_{\ell}$ exceeds a threshold $\beta_i$ and $w_i$ is the weight of the vector $a_i$.

The rest of this paper evaluates how the reliability of a Smart Grid against cascading failures can be improved by optimizing the power flow with the two optimization strategies.

IV. Evaluation

This section illustrates the experimental evaluation of the coordination mechanism for smart transformers. The system is implemented in Matlab and makes use of the MATPOWER library. The Mosek solver is used for the optimization. Three case networks are evaluated:

(i) case-30 with 4 smart transformers,
(ii) case-39 with 5 smart transformers and
(iii) case-29 with 6 smart transformers. Due to space restrictions, results are obtained for the AC power flow analysis models that is more challenging to study and relevant for all heaker modes. These case networks are chosen as they contain capacity information, the line ratings, for each line that makes the optimization with smart transformers more realistic and challenging compared to only using the tolerance parameter. The line ratings vary in case-30 to show the influence of line capacities on system reliability. Moreover, the number of transformers varies from 2 to 15 in some of the experiments to show whether more transformers result in higher reliability.

The overall system reliability of a specific case network equipped with smart transformers is evaluated under an N-1 contingency analysis with three relative metrics: (i) load shedded $L_p$, (ii) lines trimmed $L_t$ and (iii) lines utilization $U$.

Measurements are performed at every change in the status of the network, e.g. when each line fails during a cascading failure. The load shedded is measured as follows:

$$
L_p = 1 - \frac{\tilde{p}}{p}
$$

where $\tilde{p}$ and $p$ are the final and initial load served in the N-1 contingency analysis. The lines trimmed are measured as follows:

$$
L_t = 1 - \frac{\hat{L}}{L}
$$

where $\hat{L}$ and $L$ are the initial and final number of non-trimmed lines in the N-1 contingency analysis. The lines utilization is measured as follows:

$$
U = \frac{1}{L} \sum_{\ell=1}^{L} \frac{f_{\ell}}{r_{\ell}}
$$

when $f_{\ell}$ is the flow of line $\ell$ and $r_{\ell}$ is the line rating of line $\ell$ representing the line capacity.

As the positioning of the transformers is not the focus of this paper, the experiments are repeated for 10 random placements of the smart transformers. Particularly, the random placement for the case-30 takes places at the lines that do not connect the generators and affect the flow. Load shedding is performed by applying a 5% proportional reduction of the load in the respective nodes for every iteration required for convergence. A maximum number of 15 iterations are executed for convergence. The settings of the optimization strategies are chosen empirically as follows: (i) $\lambda = 0$, $\varphi_{max} = 16^\circ$, $\varphi_{max} = 7^\circ$ for STRATEGY 1 and STRATEGY 2, (ii) $\beta_1 = 0.6$, $w_3 = 1$, $\beta_2 = 0.8$, $w_2 = 10$ and $\beta_3 = 0.95$, $w_3 = 100$ for STRATEGY 2. In lines with transformers it holds that $w_3 = 1000$. When the penalty parameter is used in the experiments $\lambda > 0$, then $\lambda = 0.1$ for STRATEGY 1 and $\lambda = 1.0$ for STRATEGY 2. These number are chosen after applying a random search of the optimization space in the range $\lambda \in [0, 20]$ in increments of 0.1. Figure 1 in Supplementary Information shows an example of this process.

Results are illustrated by binning in 100 bins the values of the performance metrics derived from each line removal in the N-1 failure scenarios and computing the cumulative distribution function. The cumulative distribution functions of the 10 random placements of the smart transformers form the envelope of self-repairability of a given network. Table I, II and III in Supplementary Information provide illustrative examples of how the cumulative distribution functions are computed for each performance metric. The raw values for case-29 are illustrated in a table given that not all lines

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2 Available at [http://www.pserc.cornell.edu/matpower](http://www.pserc.cornell.edu/matpower) (last accessed: July 2016)
3 Available at [https://www.mosek.com](https://www.mosek.com) (last accessed: October 2016)
4 Case-30 and case-39 are standard IEEE benchmark networks and case-29 is a representative model of electricity transmission network in Great Britain. Available at [http://www.maths.ed.ac.uk/optimergy/NetworkData/reducedGB](http://www.maths.ed.ac.uk/optimergy/NetworkData/reducedGB) (last accessed: July 2016)
5 This network is used as follows: An initial power flow analysis detects trimmed lines. The line rating of these lines is doubled and optimal power flow is performed to derive the BASE CASE.
7 A cumulative distribution function provides an aggregate picture of the results under the N-1 contingency analysis and the different placements of smart transformers. For a given metric in the x-axis, a cumulative distribution function shifted to the left indicates low overall values of this metric, whereas, a cumulative distribution function shifted to right indicates high overall values of this metric.
can be equipped with smart transformers. The goal of the experimental evaluation is to compare the system reliability of strategy 1 and strategy 2 and make a contrast to the base case under (i) different healing modes, (ii) different networks and (iv) varied number/placements of transformers.

A. Load shedding

Figure 2 shows the envelopes of self-repairability for case-39. Strategy 2 has the lowest load shedded in all healer modes, with healer 2 having the minimal one. The penalty parameters make the strategies more stable without significantly lowering the load-shedding.

![Cumulative Distribution Function](image1)

(a) HEALER 1, $\lambda = 0, 0$

(b) HEALER 2, $\lambda = 0, 0$

(c) HEALER 3, $\lambda = 0, 0$

(d) HEALER 2, $\lambda = 0, 1, 1.0$

Fig. 2. Load shedded in case-39 with AC power flow under different healing modes and $\lambda$ values for strategy 1 and strategy 2 respectively.

With the default low load level of case-30, no significant load shedding is observed. If case-30 is stretched by decreasing the rating of the lines by 50% as shown in Figure 7, significant load shedding is performed that is much higher compared to the case-39. Strategy 2 has the lowest overall load shedded and the most stable performance given the lower area size of the envelope. The load shedded in minimal for healer 2.

Table III shows the load shedded values in case-29. It can be observed that certain line removals, e.g., lines 15 and 16, have a catastrophic effect for base case and strategy 1 in all healer modes, whereas, strategy 2 successfully mitigates the cascading failure. Given that the values of the tables indicate to a high extent the trimmed lines and lines utilization, the case-29 is omitted from the results for the rest of this section.

B. Lines trimmed

Figure 5 illustrates the lines trimmed in case-39. The envelopes confirm the highest mitigation capability of strategy 2 and healer 2. The penalty parameter has a low negative effect.

Figure 5 illustrates the lines trimmed in the performed experiments with case-30. It is also confirmed here that case-39 is a more robust network against cascading failures compared to case-30 with the chosen settings. Healer 2 has on average high, but varied, performance as indicated by the size of the envelope area.

C. Lines utilization

Figure 6 illustrates the lines utilization in case-39. Healer 3 has the lowest lines utilization because of the highest damage the network experiences as illustrated earlier. In contrast, for a higher but highly balanced utilization of the lines, Healer 2 justifies its highest resilience.
In contrast to case-39, case-30 has broader envelopes due to higher instability of the network as shown in Figure \ref{fig:case30}. The envelope of STRATEGY 2 is broader but highly shifted to the right compared to the envelope of STRATEGY 1.

D. Number and positioning of transformers

Figure \ref{fig:case30} illustrates the load shedded in case-39 under varying number of smart transformers using HEALER 3. It shows that a higher number of smart transformers results in a more effective optimization of the power flow for both strategies. The superiority of STRATEGY 2 over STRATEGY 1 is confirmed here as well with an 4.9% lower average load shedded.

Figure \ref{fig:case30} illustrates the lines trimmed in case-39 under varying number of smart transformers. It is confirmed here as well that a higher number of smart transformers result in higher reliability. STRATEGY 2 shows on average 2% lower number of lines trimmed compared to STRATEGY 1.

Table \ref{tab:case39} and \ref{tab:case30} illustrates the 10 random placements of smart transformers and the load shedded performed in case-30 and case-39 respectively. The smart transformers positions can be analyzed using topological and graph spectra metrics to design meta-heuristics for robust smart transformer placements. Such an analysis is out of the scope of this paper and it is part of future work.

E. Computational aspects

The proposed coordination scheme for smart transformers can be used by system operators to precompute mitigation
actions. This may involve running the optimization in the control centers with frequent historic scenarios of failures. Devices such as phase measurement units (PMUs) can make data accessible to control centers in real-time [21, 22] so that the mitigation actions are immediately triggered. PMUs may provide data with the rate of up to 60 [23] or even 120 samples per seconds in some industrial solutions. The feasibility of this approach is shown also in earlier work via the design of backup strategies under predefined systems conditions. The backup strategies use real-time fault analysis such as event trees [24] or controlled and optimized islanding of a power system in the order of milliseconds for its protection [25].

Given that cascading failures may be triggered by highly unexpected events, the computational cost for an entirely online optimization is evaluated in Figure 9. The two mitigation strategies are evaluated with HEALER 1, λ = 0 in three power networks. The benchmark runs in a Dell inspiron n5110 personal computer with 6GB memory, Intel(R) Core(TM) i7-2630QM CPU @ 2.00GHz with Ubuntu 15.10. The N-1 contingency analysis runs in parallel and the results show the average execution time of a single link removal. Two processing overheads are measured: (i) the optimization time that includes the underlined operations in Algorithm 1 and (ii) the other operations time that includes the rest of the operations shown in Algorithm 1.

Figure 9 confirms that the computational operations can be rapidly performed with a time overhead of a few seconds. Processing completes in less than 4 seconds in all cases. The time of the other operations is limited to a few milliseconds approaching a real-time operation. The time overhead can be further decreased by running the proposed system on a larger computer infrastructure than a personal computer. Although case-39 is the largest network, it has the lowest total time overhead as it is the most robust one against cascading failures as confirmed by the self-repairability envelopes.

Earlier work identifies slow and medium cascades that evolve in the order of minutes and seconds [26, 27]. The results confirm that the proposed scheme for self-repairable smart grids using smart transformers is highly applicable for

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**Table III**

<table>
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<th>Position</th>
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<td>7 39 38 19 30</td>
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<td>4</td>
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**Table IV**

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these cascades as parallel computations can be performed in milliseconds or in a few seconds even in a personal computer. Accessing larger and more expensive Big Data infrastructures available in control centers would be the way to deal with the larger networks that require a longer optimization. Moreover, both the optimization algorithms and the power flow calculations involve at a lower level several matrix operations that can be parallelized over GPUs with tremendous performance enhancements as studied in earlier work [28, 29].

V. COMPARISON WITH RELATED WORK

Related work on smart transformers mainly focuses on frequency control [30], voltage stability [31, 32, 33, 34], demand-response in distribution networks with multiple feeders [35] or integration on medium and low voltage microgrids [36, 37, 38]. In most of this work, a single on-load tap-changer transformer [39] is considered. In other work in which coordination of multiple transformers is considered among the transmission system operators of different countries [5], no automation and online operation is performed, e.g. optimization of allocation markets with power transformers. Moreover in the related work of [13], the corrective coordinated actions of phase shift transformers implicitly assume controlability of the overloaded lines. It is unclear how this proposed method can be applied in cascading failure scenarios that may involve load shedding or redispatch of generation.

An analytical approach to grid operation with phase shift transformers is earlier studied [18]. In contrast, this paper builds upon an analytical model and provides four contributions: (i) coordination and optimization can be performed in both AC and DC flow models. (ii) Analytical expressions formulated in matrices with all system components. (iii) Limited ranges for the phase angles are taken into account. (iv) System reliability is stretched by disruptive events that result in cascading failures. In contrast to STRATEGY 1 proposed in this paper that balances the overall utilization of the lines, this related work sets line utilization to a precomputed distribution. Monitoring is restricted to the interconnectors instead of all other influenced lines. Finally, the scope of this related methodology is mainly the optimization of border capacity between different areas. This work expands to cascading failure scenarios. In addition, post-fault precomputed corrective actions by smart transformers in case-29 are earlier introduced [5]. In contrast to this work, no automated response or an overall coordination among smart transformers is performed.

A meaningful quantitative comparison of this work with other related approaches is challenging. The reliability or repairability of power networks with smart transformers, under disturbances potentially leading to cascading failures, is usually not systematically studied. For example, a distributed multi-agent coordination of smart transformer control is introduced earlier [6]. Agents controlling physical assets of power grids communicate in real-time and control the phase angle of smart transformers. Although this approach is very promising and indicates future opportunities for distributed coordination of smart transformers, the evaluation methodology is limited to assessing individual lines that are overloaded. However, given the non-linear dynamics of AC power systems, an overall system-wide assessment is required. Decreasing the load in one line with smart transformers may result in even higher load in other lines that are usually well-balanced. The cumulative distribution functions in the evaluation methodology of Section IV show this effect. Therefore, evaluating the cost-effectiveness of any coordination mechanisms for smart transformers requires the overall assessment of the flow distribution in the network. This work exactly fills this gap.

VI. CONCLUSION AND FUTURE WORK

This paper concludes that coordinating smart transformers can provide a higher reliability in Smart Grids under disruptive events such as cascading failures or cyber-attacks. This is shown by analytically deriving two optimization strategies with which load shedded and lines trimmed are decreased while lines utilization becomes more balanced under N-1 contingency analysis. A higher number of smart transformers participating in the coordination process improves further reliability as shown with the envelopes of self-repairability in the performed experimental results.

The model proposed for self-repairable Smart Grids is extensible as other optimization strategies can be further tested as well. For example, an optimization strategy for regulating voltage stability is part of future work. Moreover an expression of the phase shift distribution factors in AC would make the model more accurate. Finally, fully decentralized sensing mechanisms studied in earlier work [40] could make the proposed model computationally scalable to large-scale network with all components interacting and self-regulating power flow in a collective fashion.

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